Pion-Nucleon Physics and the Polarizabilities of the Nucleon

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Abstract

I present recent results regarding the influence of pion-nucleon and nucleon-resonance physics on the polarizabilities of the nucleon as measured in Compton scattering.

INTRODUCTION

In this workshop on pion-nucleon physics I am going to talk about Compton scattering off the nucleon, which may seem to be a strange topic to be presented here. However, I want to convince you that pion-nucleon and also $\Delta(1232)$ dynamics are essential for understanding the results from low energy Compton scattering–by this I mean photon energies in the c.m. frame of less than 100 MeV. Let us start with a very simple picture of the nucleon: A point particle with spin 1/2 and an anomalous magnetic moment κ. In the 1950s Powell has already calculated the cross section for this model and it turns out it describes the experimental data quite well for photon energies up to 50 MeV.

If one increases the energy of the incoming photon beam one starts seeing deviations from the simple Powell predictions, as one is picking up sensitivity to the internal structure of the nucleon. In the past few years we have learned that this *low energy structure* of the nucleon can be described very well in terms of virtual pion excitations around an unresolved spin 1/2 nucleon, together with some contributions from nucleon resonances. The most precise method to calculate these structure effects to this date is heavy baryon chiral perturbation theory (HBChPT), for a recent review see [1]. If you wish, you can say that HBChPT is an effective field theory with systematic power counting that allows for a precise calculation of the "pion cloud" of the nucleon. In this presentation I will briefly outline the low energy structure of the Compton amplitude, define the polarizabilities of the nucleon and present new predictions obtained in a ChPT framework with explicit pion, nucleon and delta degrees of freedom. For details and more references see [2,3].

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COMPTON SCATTERING AT LOW ENERGIES

Assuming invariance under parity, charge conjugation and time reversal symmetry the general amplitude for Compton scattering off a proton ($\gamma\ p \to \gamma'\ p'$) can be written in terms of 6 structure dependent functions $A_i(\omega, \theta),\ i = 1..6$, with $\omega = \omega'$ denoting the photon energy and θ being the scattering angle:

$$\begin{aligned} T_{cms} = & A_1(\omega,\theta)\ \vec{\epsilon}^{*\prime} \cdot \vec{\epsilon} + A_2(\omega,\theta)\ \vec{\epsilon}^{*\prime} \cdot \hat{k}\ \vec{\epsilon} \cdot \hat{k}' + A_3(\omega,\theta)\ i\vec{\sigma} \cdot (\vec{\epsilon}^{*\prime} \times \vec{\epsilon}) \\ & + A_4(\omega,\theta)\ i\vec{\sigma} \cdot (\hat{k}' \times \hat{k})\vec{\epsilon}^{*\prime} \cdot \vec{\epsilon} + A_5(\omega,\theta)\ i\vec{\sigma} \cdot [(\vec{\epsilon}^{*\prime} \times \hat{k})\vec{\epsilon} \cdot \hat{k}' - (\vec{\epsilon} \times \hat{k}')\vec{\epsilon}^{*\prime} \cdot \hat{k}] \\ & + A_6(\omega,\theta)\ i\vec{\sigma} \cdot [(\vec{\epsilon}^{*\prime} \times \hat{k}')\hat{\epsilon} \cdot \hat{k}' - (\vec{\epsilon} \times \hat{k})\vec{\epsilon}^{*\prime} \cdot \hat{k}] \end{aligned} \tag{1}$$

Here $\vec{\epsilon}, \hat{k}\ (\vec{\epsilon}', \hat{k}')$ are the polarization vector, direction of the incident (final) photon while $\vec{\sigma}$ represents the (spin) polarization vector of the nucleon. One now performs a low-energy expansion of the 6 independent functions $A_i(\omega, \theta)$ in powers of the photon energy ω. For the case of a proton target of mass M_N with anomalous magnetic moment $\kappa^{(p)}$ one finds

$$A_1(\omega,\theta)_{cms} = -\frac{e^2}{M_N} + 4\pi\left(\alpha_E^{(p)} + \cos\theta\ \beta_M^{(p)}\right)\omega^2 - \frac{e^2}{4M_N^3}(1-\cos\theta)\,\omega^2 + \dots \tag{2}$$

$$A_2(\omega,\theta)_{cms} = \frac{e^2}{M_N^2}\omega - 4\pi\beta_M^{(p)}\omega^2 + \dots \tag{3}$$

$$\begin{aligned} A_3(\omega,\theta)_{cms} = & \left[1 + 2\kappa^{(p)} - (1+\kappa^{(p)})^2\cos\theta\right]\frac{e^2}{2M_N^2}\omega - \frac{(2\kappa^{(p)}+1)e^2}{8M_N^4}\ \cos\theta\ \omega^3 \\ & + 4\pi\left[\gamma_1^{(p)} - (\gamma_2^{(p)} + 2\gamma_4^{(p)})\cos\theta\right]\omega^3 + \dots \end{aligned} \tag{4}$$

$$A_4(\omega,\theta)_{cms} = -\frac{(1+\kappa^{(p)})^2 e^2}{2M_N^2}\omega + 4\pi\gamma_2^{(p)}\omega^3 + \dots \tag{5}$$

$$A_5(\omega,\theta)_{cms} = \frac{(1+\kappa^{(p)})^2 e^2}{2M_N^2}\omega + 4\pi\gamma_4^{(p)}\omega^3 + \dots \tag{6}$$

$$A_6(\omega,\theta)_{cms} = -\frac{(1+\kappa^{(p)}) e^2}{2M_N^2}\omega + 4\pi\gamma_3^{(p)}\omega^3 + \dots \tag{7}$$

The leading terms in the 6 structure functions are completely model-independent and coincide with the old low energy theorems of current algebra. The "real" structure dependence beyond the anomalous magnetic moment starts at sub-leading order in the ω expansion and the associated 6 polarizabilities $\alpha_E,\ \beta_M,\ \gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_4$ cannot be determined by symmetry considerations. In *unpolarized* Compton scattering the electric polarizability α_E and the magnetic polarizability β_M describe the leading structure dependent effects and account for the deviation of the cross section from the Powell result. The most recent fits yield [4]

$$\alpha_E^{(p)} = (12.1 \pm 0.8 \pm 0.5) \times 10^{-4}\,\text{fm}^3, \qquad \beta_M^{(p)} = (2.1 \mp 0.8 \mp 0.5) \times 10^{-4}\,\text{fm}^3, \tag{8}$$

indicating that the nucleon is a rather "stiff" object that cannot easily be deformed in the electric and magnetic field of the incoming and outgoing photon.

While the linear response to external electric and magnetic fields $(\vec{E}, \vec{B})$ for a classical (macroscopic) object is uniquely determined by the 2 polarizabilities α_E, β_M, the nucleon due to its extra spin 1/2 degree of freedom has 4 additional response parameters γ_i in external

$\vec{E}, \vec{B}$ fields, commonly called the "spin-polarizabilities" of the nucleon [5]. All 6 structure dependent parameters are intrinsic properties of the nucleon and their determination in Compton scattering therefore amounts to a test of (low energy) QCD.

There exists a long history of experiments trying to determine α_E, β_M, whereas the 4 spin-polarizabilities have only recently attracted the attention of experimentalists, as one requires polarized photon sources in addition to polarized targets and has to measure over a wide range of scattering angles θ in order to extract the γ_i contributions. In the absence of double-polarization experiments one has nevertheless tried to obtain some estimates of particular linear combinations of the 4 γ_i from multipole analyses in the single pion production region and unpolarized Compton scattering in the backward direction. This is not the place to comment in detail on these "experimental" determinations, but I want to express a strong caveat that the quoted errors could be severely underestimated due to strong model-dependencies of the extraction process. Keeping this in mind, the current knowledge of spin-polarizabilities from *unpolarized* data reads [6]

$$\gamma_0^{(p)} = \gamma_1 - \gamma_2 - 2\gamma_4 \approx -1.34 \times 10^{-4}\ \mathrm{fm}^4\ , \tag{9}$$
$$\gamma_\pi^{(p)} = \gamma_1 + \gamma_2 + 2\gamma_4 \approx -\,(28.0 \pm 2.8 \pm 2.5) \times 10^{-4}\ \mathrm{fm}^4\ . \tag{10}$$

Further details on the current experimental situation can be found in [3]. The huge numerical difference between γ_0 and γ_π can be understood if one analyses the underlying physics using ChPT.

THE PHYSICS BEHIND THE POLARIZABILITIES

In 1992 it was found in a $\mathcal{O}(p^3)$ HBChPT calculation [7] that ChPT can very nicely explain the magnitude of both α_E and β_M as being dominated by πN loop effects. According to this interpretation the only structure of the nucleon a low energy photon resolves when undergoing Compton scattering would therefore be given by the nucleon's "pion-cloud", in marked contrast to analyses using dispersion relations [8]. A subsequent $\mathcal{O}(p^4)$ calculation [9] proved that there are indeed only small corrections to the $\mathcal{O}(p^3)$ result, yielding

$$\alpha_E^{(p)}|_{O(p^4)} = (10.5 \pm 2.0) \times 10^{-4}\ \mathrm{fm}^3, \qquad \beta_M^{(p)}|_{O(p^4)} = (3.5 \pm 3.6) \times 10^{-4}\ \mathrm{fm}^3. \tag{11}$$

We note the agreement with current experimental results (Eq.8), but also that there exists a considerable theoretical uncertainty. The main reason for this uncertainty is the first nucleon resonance $\Delta(1232)$, which in HBChPT can only be included via counterterms, i.e. is taken to be infinitely heavy compared to the nucleon. Recently a systematic formalism has been developed to include the delta as an explicit degree of freedom in ChPT, called the "small scale expansion" [10]. Herein one organizes the calculation in powers of ϵ, which denotes either a soft momentum, the pion mass or the nucleon-delta mass splitting. The 6 polarizabilities of the nucleon have been calculated to $\mathcal{O}(\epsilon^3)$ within this approach, taking into account all contributions arising from πN loops, Δ-pole graphs, $\pi\Delta$ loops and neutral pion exchange via the anomalous $\pi^0\gamma\gamma$ vertex. The pertinent Feynman diagrams and a discussion of the technical aspects regarding calculations in this formalism can be found in [2]. Using a new determination of the relevant $N\Delta$ coupling parameters one finds the spin-independent polarizabilities

$$\alpha_E^{(p)}|_{O(\epsilon^3)} = [12.2(\mathrm{N}\pi - \mathrm{loop}) + 0(\Delta - \mathrm{pole}) + 4.2(\Delta\pi - \mathrm{loop}) + 0(\mathrm{anom.})] \times 10^{-4}\,\mathrm{fm}^3\ , \quad (12)$$

$$\beta_M^{(p)}|_{O(\epsilon^3)} = [1.2(\mathrm{N}\pi - \mathrm{loop}) + 7.2(\Delta - \mathrm{pole}) + 0.7(\Delta\pi - \mathrm{loop}) + 0(\mathrm{anom.})] \times 10^{-4}\,\mathrm{fm}^3\ . \quad (13)$$

A quick glance at these results shows that the $\mathcal{O}(\epsilon^3)$ calculation is not able to reproduce the experimental results. In particular, the large diamagnetic "recoil" contribution of the πN loops in the case of β_M is only entering at $\mathcal{O}(p^4)$ in HBChPT [9] and thus necessitates a $\mathcal{O}(\epsilon^4)$ calculation for a cancelation of the large paramagnetism of the Δ-pole contribution in Eq.13. Though numerically discouraging at this order, the solution to this old problem in calculations of β_M is thus known from the $\mathcal{O}(p^4)$ calculation and is expected to work as well at $\mathcal{O}(\epsilon^4)$ in the small scale expansion. The more "troubling" aspect of Eqs.12f is actually the large contribution from the $\pi\Delta$ continuum to α_E. In HBChPT it is very common to subsume pole contributions from nucleon resonances in counterterms, but there is no agreement in the chiral community yet how one would include πN^* or $\pi\Delta^{(*)}$ loop effects in counterterms at a given order. Usually these effects are quite small and can be safely neglected, but the $\mathcal{O}(\epsilon^3)$ calculation of α_E shows a strong counterexample. A future $\mathcal{O}(\epsilon^4)$ calculation will therefore shed more light on the underlying physics in α_E and the issue of resonance saturation of counterterms in the baryon sector in general.

I now move on to discuss the physics of the spin-polarizabilities. In HBChPT they had been calculated to $\mathcal{O}(p^3)$ [1], but it was quickly realized that $\Delta(1232)$ could give large corrections. Now, unlike the case of α_E, β_M where Δ-pole contributions could be incorporated at $\mathcal{O}(p^4)$ via counterterms, in the case of the spin-polarizabilities one would have to go to $\mathcal{O}(p^5)$, i.e. 2-loop, to saturate the counterterms with delta exchange in a *complete* calculation. I am sure you know that 2-loop calculations in the baryon sector are outside today's ChPT technology, so the spin-polarizabilities were "forgotten" for a while except for some occasional phenomenological modeling. With the advent of the "small scale expansion" method the situation finally changed–it is now possible to systematically calculate the effects of $\Delta(1232)$ on the γ_i with 1-loop technology. I present here the results of a recent $\mathcal{O}(\epsilon^3)$ calculation [3]:

$$\gamma_1^{(p)} = [4.6(\mathrm{N}\pi - \mathrm{loop}) + 0(\Delta - \mathrm{pole}) - 0.2(\Delta\pi - \mathrm{loop}) - 22(\mathrm{anom.})] \times 10^{-4}\,\mathrm{fm}^4 \quad (14)$$

$$\gamma_2^{(p)} = [2.3(\mathrm{N}\pi - \mathrm{loop}) - 2.4(\Delta - \mathrm{pole}) - 0.2(\Delta\pi - \mathrm{loop}) + 0(\mathrm{anom.})] \times 10^{-4}\,\mathrm{fm}^4 \quad (15)$$

$$\gamma_3^{(p)} = [1.2(\mathrm{N}\pi - \mathrm{loop}) + 0(\Delta - \mathrm{pole}) - 0.1(\Delta\pi - \mathrm{loop}) + 11(\mathrm{anom.})] \times 10^{-4}\,\mathrm{fm}^4 \quad (16)$$

$$\gamma_4^{(p)} = [-1.2(\mathrm{N}\pi - \mathrm{loop}) + 2.4(\Delta - \mathrm{pole}) + 0.1(\Delta\pi - \mathrm{loop}) - 11(\mathrm{anom.})] \times 10^{-4}\,\mathrm{fm}^4 \quad (17)$$

Note that 3 of the 4 polarizabilities are dominated by neutral pion exchange coupled with the anomalous $\pi^0\gamma\gamma$ vertex. In addition to this well-understood contribution there are strong interference effects between πN loops and $\Delta(1232)$ pole graphs, whereas the $\pi\Delta$ continuum shows very little influence in the spin-sector. Comparing with the known "experimental" determinations Eqs.9f one finds

$$\gamma_0^{(p)}|_{O(\epsilon^3)} = +2.0 \times 10^{-4}\,\mathrm{fm}^4, \qquad \gamma_\pi^{(p)}|_{O(\epsilon^3)} = -37.2 \times 10^{-4}\,\mathrm{fm}^4, \quad (18)$$

which reproduces the dramatic difference in size between these 2 linear combinations of spin-polarizabilities (anomaly contributions cancel *exactly* in γ_0 and are *maximal* in γ_π), but are not in very good numerical agreement. As mentioned before, the results Eqs.9f were extracted from *unpolarized* experiments and should be checked in a planned [11] double-polarization experiment, whereas on the theoretical side one has to study possible $\mathcal{O}(\epsilon^4)$ corrections to Eqs.14ff to judge the convergence of the perturbation series.

CONCLUSION

I have presented recent results for the 6 polarizabilities of the proton calculated to $\mathcal{O}(\epsilon^3)$ in the "small scale expansion" of ChPT which modify the simple picture of the polarizabilities as just being a πN loop effect in HBChPT. $\mathcal{O}(\epsilon^4)$ calculations are called for to get a better understanding of the underlying physics , whereas on the experimental side a new experimental program has to start in order to determine the poorly known spin-polarizabilities.

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